The Geometrical Interpretation of the Simple Objects of Wittgenstein's *Tractatus*

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July 17, 2015

Introduction

In this paper, I will examine Leonard Goddard and Brenda Judge's 1982 book *The Metaphysics of Wittgenstein's Tractatus*. I will focus on the second chapter of their book, where they offer a geometric interpretation of the metaphysics of the Tractatus. Under this interpretation, Wittgensteinian simple objects are interpreted as geometric points, and states of affairs as geometric figures on the plane. Several intriguing questions are raised by this geometric interpretation. Did geometry provide Wittgenstein an inspiration or guiding metaphor for his metaphysics? What is the value of a formal model for the Tractarian metaphysics? Is Goddard and Judge's geometric model a reasonable model of Wittgensteinian simples? In what ways can Goddard and Judge's geometric interpretation be improved? In this paper, I plan to give an exposition of the Goddard and Judge geometric model, and to address these and related questions.

The plan of the this paper will be as follows. In the first section, I will overview some of the previous interpretations of Wittgensteinian simples objects, and explore the question whether Wittgenstein was using geometry as an inspiration of metaphor for his metaphysics. In the second section, I will

address the question of what value a consistency model of the Wittgensteinian metaphysics has, focusing on how a consistency model is a barrier to the resolute reading of the *Tractatus*. In the third section, I will describe the Goddard and Judge geometric interpretation, offer some general concerns any model of metaphysical simples faces, and give a critical analysis of the Goddard and Judge interpretation. In the fourth and final section of this paper, I will describe how the Goddard and Judge geometric model can be extended to address some of its defects.¹

1 Previous Views on Simple Objects

There have been a number of previous interpretations of Wittgenstein's simple objects. In The Philosophy of Logical Atomism, Bertrand Russell argues that things like sense datum, properties of sense datum, and subjects are all types of simple objects [8]. Expanding on the Russellian view, Jaako Hintikka maintains that Wittgensteinian simples are to be interpreted as something like pure sense datum, for instance specks of color. In his 2006 book Wittgenstein's World of Mechanics, Gerd Grashoff puts forward a physicalist theory of simples, one in which Wittgensteinian simples are to be interpreted as mass particles [6]. It has even been suggested that the world itself might be the only example of a simple object. Against this project of giving examples of simples, Hidé Ishiguro argued that Wittgensteinian simple objects must be unlike anything in the empirical world. These objects do not have properties through which we can refer to them. Rather, on her view, reference to these objects is only possible with language in place. Simple names, she argues, act like "p" in "let p be the center of the sphere S," and are "dummy variables" [7].

Wittgenstein himself had a number of intriguing pre-Tractatus thoughts

¹In this paper, when I refer to Wittgenstein and his metaphysics, I will be referring to the early Wittgenstein and the metaphysics of his *Tractatus* and *Notebooks* period, unless otherwise noted.

about possible examples of simple objects. In his *Notebooks*, he asks, "Is a point in our visual field a simple object, a thing?" (September 3, 1914). However less than a year later, he seems to have serious doubts about finding examples of simple objects. He writes, "But it also seems certain that we do not infer the existence of simple objects from the existence of particular simple objects, but rather know them by description, as it were as the end-product of analysis, by means of a process that leads to them" (May 23, 1915). For him, it seems, simple objects are the necessary product of an analysis of complex objects, and not to be found somewhere out in the world.

There is no mention by Wittgenstein of geometric points as possible examples of simple objects in the *Notebooks* or the *Tractatus*. The evidence provided by Goddard and Judge that Wittgenstein was employing geometry as a metaphor for his metaphysics lies in 3.411 "In geometry and logic alike a place is a possibility" and in Wittgenstein's references to Heinrich Hertz's *The Principle of Mechanics Presented in a New Form*, which Goddard and Judge argue is highly geometric in is presentation. I take this evidence to be weak, in that 3.411 is an isolated proposition that occurs outside the main exposition of Wittgenstein's metaphysics in sections 1 and 2. Reference to Hertz's work, though passing, is intriguing, and it would be interesting to explore how much of an influence the work had on the early Wittgenstein. Based though on what is found in the extant *Notebooks* and the *Tractatus*, I conclude that Wittgenstein was neither thinking seriously of geometric points as simple objects, nor was he employing geometry as an inspiration or guiding metaphor for his metaphysics.

All of this leaves open the possibility of a contemporary interpretation of Wittgensteinian objects as mathematical entities, aside from from the what the influences of Wittgenstein were. In particular, an example of a simple object might be a mathematical entity that is a logical necessity of an analysis of other mathematical entities. Goddard and Judge proposed exactly this, interpreting Wittgensteinian simple objects as geometric points on the plane. As far as I know, there have been no attempts to model Wittgensteinian simples as mathematical objects previous to Goddard and Judge's work, and so their work stands as an attractive interpretation of Wittgensteinian simples to analyze.

2 The Value of a Consistency Model

The reader may wonder at this point, what is the value of a model of the metaphysics of the Tractatus? After all, the reader may press, we want to know what is true, not just merely consistent. A consistency model of the metaphysics, I will argue, is nonetheless valuable in our understanding of the Tractatus. In particular, such a model can be taken as an argument against the resolute reading of the *Tractatus* by Cora Diamond, Warren Goldfarb and others. On the resolute reading, Wittgenstein's metaphysical propositions neither say nor show anything, they are just non-sense. In the 2.01's of the Tractatus, Wittgenstein is building a metaphysics that is designed to fail. The system is a myth, but in trying to understand the myth, we see how it and all other metaphysical stories fall. The value of reading the *Tractatus* then is a dialectical exercise, in that we become better philosophers when we grapple with the text. These interpreters hold that there is an esoteric reading to the Tractatus, or a deeper authorial intent besides setting a description of a metaphysics and how language relates to it. The metaphysics of the Tractatus, on this reading, is incoherent.

A Goddard and Judge style model of Wittgenstein's metaphysics though would undercut the resolute reading. First, if the idea that Wittgenstein was taking geometry as a model for his metaphysics is correct, this looks like further evidence that Wittgenstein really was trying to setup a genuine metaphysics in the Tractatus. Resolute readers are faced with the biographical question of what Wittgenstein meant when he said "I used to believe there was a connection of world and language." An examination of the plausibility of the Goddard and Judge model can help us examine how seriously Wittgenstein was taking geometry as a model of his metaphysics, and how serious he was in building a metaphysics in the Tractatus.

Secondly the resolute reading holds that the metaphysics of the 2.01's is inconsistent, and that discovering this inconsistency is what drives us down the dialectic path that Wittgenstein intends. Writing of metaphysical sentences, Goldfarb summarizes the resolute position: "We think we have some understanding of such sentences, perhaps by dint of psychological associations we have with them or mental images they call to mind, abetted by the sentences' having apparent logic form parallel to unproblematic sentences. We are, however, meant to *interrogate* that understanding, particularly as we read on in the text of learn more of the procedures that Wittgenstein is trying to formulate. At some point when we carry the interrogation far enough, the incoherence of the original sentences will become manifest."

But a consistency model like the geometric interpretation would show that the sentences of the 2.01's are not incoherent. To borrow a concept from mathematical logic, if we can exhibit a model of some sentences, those sentences are consistent, and if a theory can prove that a model of some sentences exist, then the theory proves that the sentences are consistent. So if we can show that two-dimensional geometry models the metaphysical sentences of the Tractatus, under the interpretation of objects as geometric points and facts as geometric figures on the plane, we would have shown that the metaphysical sentences are not inconsistent with respect to first-order logic. I will be treating the notions of a model and interpretation as essentially the same, based on the observation that a model is as an interpretation of the non-logical constants of a formal language as some objects.

One must be careful that the sentences of the *Tractatus* are not given as statements in a formal system, and the Goddard-Judge model is not presented as a model in the sense of model theory. But this argument of consistency by demonstrating a model still seems like a barrier for the resolute reader to overcome. In 6.53, a passage often used in support of the resolute position, Wittgenstein writes "The correct method of philosophy would really be the following: to say nothing except what can be said, i.e. propositions of natural science–i.e. something that has nothing to do with philosophy– and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give meaning to certain signs of his propositions." Against this, a consistency model like the geometric interpretation would be exactly a complete determination of all the signs in the propositions of the Tractarian metaphysics.

Someone like Goldfarb or Diamond might push back, saying that though they are not inconsistent, the metaphysical sentences are nonetheless incoherent. Coherence, on this counter-argument, is a stronger property than consistency. But I cannot see how to make this difference between consistency and coherency precise or exact. Another counter-argument might be that the sentences are indeed inconsistent with respect to a stronger logic than first-order logic, or that they are inconsistent with the later passages of the Tractatus. On the first approach, I would hesitate that Wittgenstein would endorse something other than first-order logic as the right formalization of logical thinking. On the second approach, I think that the inconsistency of the metaphysics with the rest of the *Tractatus* is the strongest counterargument for the resolute readers. I think exploring whether an expansion of the geometric interpretation can render the metaphysics consistent with larger parts *Tractatus* is one of the most interesting and important further directions to take.

3 The Geometric Interpretation and its Defects

On Goddard and Judge's geometric interpretation, an "object," "entity," or "thing" in the *Tractatus* is to be interpreted as a "geometric point" in the model. When the context is clear, "geometric" is dropped, so that a "geometric point" becomes a "point". An "atomic fact" translates to an "actual geometrical line segment" in the model. A "possible atomic fact" is a "line segment." A "fact" is an "actual figure on the plane." A "state of affairs" is a "line segment or figure". "Logic" and "logical" become "geometry" and "geometrical." "Logical space" is interpreted as "geometric space." "Logical scaffolding" becomes "a coordinate system" or "geometric coordinates."

Apart from any particular defects that Goddard and Judge's geometric model might have, I want to outline some general concerns that readers may have about any proposed model of metaphysical simples. The first general concern is whether any formal model can give us examples of objects which are propertyless. Wittgenstein gives no examples of his objects in the Tractatus. Rather, by 1918, he seems to think of simples as theoretical endpoints to an analysis of what objects must be composed of. One could argue that neither could he give us examples of propertyless simples. To present us an example of an object, such as a sense datum like color speck, or a property such as redness, would be to seemingly present something which has some property. But an object with properties could not be a Wittgensteinian simple.

A possible response to this, to use an idea from model theory, is to give a model together with a restricted language. The objects might be propertyless with respect to the restricted language, yet nonetheless have properties with respect to an enriched language. In the context of the geometric interpretation, the base language could be a formal language with only geometric constants and relations such as "points a and b are equidistant from c," and "point y lies between points x and z," while the enriched language could be something like the language of a stronger formal theory of mathematics, or a non-formal language like ordinary mathematical discourse. I think that this approach of a base language in which the simples are propertyless, and a meta-language in which we can glean meaningful information about them, is a useful approach to take. And for readers of the *Tractatus* who disagree with the resolute reading, this might be exactly a fruitful formalization of the difference between what is said and what is implied or shown in the text.

The second general concern is whether we could ever tell that our model is applicable to the metaphysics we are trying to describe. In analyzing complex objects into their simple components, how do we know the we have reached the end of analysis when it comes to metaphysics? Is it the end of our analysis or the end of any analysis? These sorts of general concerns about the possibility of metaphysics were raised by Wittgenstein in his *Notebooks*: "But it is clear that components of our propositions can be analysed by means of a definition, and must be, if we want to approximate to the real structure of the proposition. At any rate, then, there is a process of analysis. And can it not now be asked whether this process comes to an end? And if so: What will the end be?" (September 15, 1915). If we can not even be sure whether our analysis of metaphysics is complete, then one may very well wonder how can interpreting our analysis of metaphysics with something like geometric points can be started or judged a success of failure. On this point, I do not have much to say other than that this is a skeptical worry to be cognizant of. Our modeling enterprise, if it is to get off the ground, seems to require that we have an analysis of simple objects which we recognize as sufficiently developed.

The geometric interpretation of objects as points is intended by Goddard and Judge so that the points and their combinations satisfy their interpretation of Wittgenstein's metaphysics. In particular, they argue, Wittgenstein's simples are propertyless, simple, and imperceptible. These objects combine into what they call atomic facts which do have properties, but which are still imperceptible. I largely agree with Goddard and Judge's understanding of Wittgenstein's simple objects. When Wittgenstein writes "In a manner of speaking, objects are colourless" (2.0232), I take it that Wittgenstein also takes objects as lacking any other properties. Objects are non-complex according to him, for instance when he writes "Objects are simple" (2.02). That they are imperceptible, we have "A speck in the visual field, though it need not be red, must have some colour Notes must have *some* pitch, objects of the sense of touch *some* degree of hardness, and so on." In this passage, I am taking him to be saying that anything we might perceive must have some properties, and since objects have no properties by 2.0232, objects cannot be perceptible. Hence I think there is strong textual evidence to support the Goddard and Judge characterization of Wittgensteinian simples as propertyless, simple, and imperceptible.

I disagree however with Goddard and Judge's understanding of how Wittgenstein's objects combine into states of affairs. Firstly, they stress a robust difference between atomic states of affairs on the one hand, and states of affairs, or what Wittgenstein calls "Tatsachen," on the other. States of affairs, when not prefixed by atomic, on the other hand, are taken to be non-atomic. In their description of Wittgensteinian simples, a prototypical atomic states of affairs are two simples in one connexion. A prototypical state of affairs for Goddard and Judge are everyday facts like "There is a car next to the truck." This difference shows up in their model: atomic states of affairs are interpreted as lines, while states of affairs are interpreted as figures. But I think that this distinction between atomic and non-atomic facts is just not supported by the Tractatus, and I just do not see anything in the early Wittgenstein that argues for the distinction. I think that their geometric interpretation then misrepresents the early Wittgensteinian metaphysics on this important point.

I think though that there is a way in which the distinction between atomic

and non-atomic facts is a tempting mistake, and a way in which it can help us see why Goddard and Judge read Wittgenstein in this way. It is essentially a misapplication of a linguistic distinction onto a metaphysical one via the Tractarian idea of mirroring. For Wittgenstein, there indeed is a robust difference between an elementary proposition, *Elementarsatz*, and a proposition, *Satz*. In proposition 5, we have, "A proposition is a truth-function of elementary propositions. (An elementary proposition is a truth-function of itself.)" Here I take Wittgenstein to be asserting that each proposition can be decomposed truth-functionally into basic elements, the elementary propositions, and drawing a distinction between simple and molecular propositions. Further support for the distinction between *Elementarsatz* and *Satz* follows in 5.01: "Elementary propositions are the truth arguments of propositions." Here I take him to be saying that whether a proposition is true or not is determined by the truth or falsity of its constituent elementary propositions.

Wittgenstein also appears to hold the idea that language is a reflection, or a mirror-image of, metaphysics. We have in 4.121 that "Propositions cannot represent logical form: it is mirrored in them. . . . Propositions *show* the logical form of reality. They display it." And elementary propositions, according to Wittgenstein, refer directly to state of affairs: "The simplest kind of proposition, an elementary proposition, asserts the existence of a state of affairs" (4.21). It is tempting then to combine Wittgenstein's distinction between elementary propositions and propositions with the idea of mirroring between language and metaphysics to conclude that there is robust difference between the states of affairs that elementary propositions and the states of affairs that propositions refer to. To spell it out, the argument would be that the important distinction between elementary propositions and propositions is mirrored in an important distinction between "elementary" states of affairs and non-elementary, or what we might call "molecular" states of affairs.

This is exactly what Goddard and Judge seem to think when they write, "[Objects] combine to form atomic facts which, though still unperceivable, do have properties; and that what we call facts at the perceptual level -e.g.that the table is next to this chair – are simply assemblages of, though not logical constructions form, these unperceivable atomic facts." But I do not think that this is correct. First Wittgenstein says that elementary propositions assert the existence of states of affairs, saying nothing about them referring to supposed atomic or elementary states of affairs. Elementary propositions need not refer to just the simplest kinds of states of affairs, as Goddard and Judge seem to assume. Secondly I do not think that mirroring works in the way described in the paragraph above. 4.121 says that propositions themselves mirror reality, but nothing about whether the difference between elementary propositions and propositions is mirrored in reality. In other words, linguistic propositions mirror reality, but the difference between Elementsatz and *Satz* need not be mirrored in reality. While it is true that we can hope to decompose a proposition into its elementary propositions, see what states of affairs these elementary propositions assert the existence of, and then say that the original proposition refers to something like a combination of those states of affairs. But the connection between propositions and states of affairs in such a construction is only indirect.

4 Extending the Geometric Interpretation

In this section, I plan to raise another set of issues that I see in the Goddard-Judge interpretation, though ones which I believe are addressable, and outline how the model may be extended to deal with these issues. One difficulty for the geometric model is whether Goddard and Judge propose their translation as an embedding or as an isomorphism of Wittgensteinian simple objects into two-dimensional geometry. If they are claiming that the metaphysics is isomorphic to the geometric plane, then this seems to commit them to interpreting Wittgenstein as holding that there are uncountably infinitely many simples object, one for every point in the plane. But Wittgenstein does not say anything in the *Tractatus* regarding the question of how many simple objects there might be. Furthermore in his *Notebooks*, he writes against the idea that a decomposition into infinite parts is the type of analysis we are after: "Let us assume that every spatial object consists of infinitely many points, then it is clear that I cannot mention all these by name when I speak of that object. Here then would be a case in which I *cannot* arrive at the complete analysis in the old sense at all; and perhaps just this is the usual case" (page 62). He states that an "*infinitely* complex situation seems to be a chimera" (page 50). Then it seems that the geometric interpretation cannot be an isomorphism between the metaphysics of the *Tractatus* and the geometric plane. I would amend the geometric model by allowing the mapping from objects to points to not just be an isomorphism, but an embedding. Wittgensteinian simples are mapped to some proper subset of the geometric plane, so that the objects are isomorphic to this subset of the plane. An advantage of thinking of the mapping as an embedding is that, no matter whether we conclude that the Wittgensteinan metaphysics is finite or infinite, we can nonetheless still use the geometric interpretation by modifying which embedding of the objects into points we take.

Another vexing issue for the Goddard and Judge geometric model is to what states of affairs logical combinations of propositions refer to, given that we know where the constituent propositions refer to. The simplest illustration of the difficulty that the model faces is with negation. On the Goddard and Judge interpretation, a state of affairs that is the connexion of two simples sand t is translated as a line between two points a and b. Assume that we have a proposition p referring to the state of affairs. Though Goddard and Judge do not make clear what p is meant to be translated to, we can propose two options. It can be translated as the geometric expression "the line between a and b," or as the geometric existence statement "the line between a and b exists". In either case, we may wonder, what should be the geometric interpretation of $\neg p$?

I can see two approaches one can take to extend the geometric interpretation. The first follows Wittgenstein when he asserts that both p and $\neg p$ refer to, or correspond to, the same state of affairs: "The propositions 'p' and $(\neg p)$ have opposite sense, but there corresponds to them one and the same reality" (4.0621). If we think that p is translated as "the line between a and b exists", then on the geometric model then, $\neg p$ would be the proposition "the line between a and b does not exist." Both p and $\neg p$ would refer to the same line, but would have opposite senses. The proposition p would have the sense that the line exists, while the proposition $\neg p$ would have the sense that the line does not exist. There is one problem with this approach for Goddard and Judge though. They maintain that on the geometric plane, it makes no sense to talk about the non-existence of geometric lines. Lines between two points always exist, on their conception. To account for the difference between potential states of affairs and subsistent states of affairs, they use infinite lines as the translation of potential states of affairs, while they use finite line segments as the translation of subsistent states of affairs. To me this seems like an unappealing ad-hoc correction. Furthermore this conception is unappealing to me since it says nothing about the important Wittgensteinian distinction between potential and subsistent states of affairs. What the translation of a non-subsistent state of affairs should be is left entirely unspecified in the Goddard and Judge interpretation.

One way to solve this issue is to move to a colored geometric plane, and to separate whether p holds from the existence or non-existence of some line. Instead of saying that p is "the line between a and b exists", we can have p translated as "the line between a and b is colored green." Then $\neg p$ could be translated as "the line between a and b is colored red." The specific colors here, of course, do not matter, as long as they are different. Such a move would side-step any issue of the existence or non-existence of lines in the plane. Rather, we could allow Goddard and Judge that lines always exist, while modeling that states of affairs subsist or not by mapping them to different colors. Furthermore, this would not run afoul of the fact that the simple objects are to be propertyless for Goddard and Judge. On their interpretation, they indeed allow lines to have properties.

The second approach that I can see is to hold that the geometric lines are directional. We might think that the vector model would be able to account for the negation of a proposition. If the proposition p is translated as the "the line between a and b with direction from a to b," then p would be translated as "the line between a and b with direction from b to a." The two propositions would correspond to the same reality in the geometric model, if we translate "reality" as the line segment between a and b, but their senses would be different, if we interpret "sense" as the direction of the line segment. If, on the other hand, we think that p should be translated as the existence statement "the vector from a to b exists," then $\neg p$ would be translated as "the vector from b to a exists." However there is a large issue for the vectorized approach to face. If the proposition p is "the vector from a to b exists", then the fact that p subsists in the model says nothing about whether $\neg p$ subsists or not. On the geometric model, if p subsists, then $\neg p$ "the vector from b to a exists" might very well subsist as well. But surely Wittgenstein would not want to say that the contradiction p and $\neg p$ could subsist. One approach to address this for the vector model would be to simply stipulate that once a vector exists on the plane, then its opposite directioned vector cannot. But I think this would be too ad-hoc to be convincing.

Conclusion

In this paper, I have aimed to provide an exposition and critical analysis of Goddard and Judge's geometric interpretation of Wittgenstein's simple objects, and to explore some questions surrounding Wittgensteinian simple objects and attempts to give a formal model of them. Based on the weakness of existing textual evidence, I conclude that Wittgenstein did not seem to be thinking of geometric points as examples of simple objects, nor was he using geometry as a guiding metaphor for his metaphysics. I have argued that a successful model of Wittgenstein's simples would present an obstacle to the resolute reading of the Tractatus, in particular, the claim that the Tractarian metaphysics is incoherent. I have argued that Goddard and Judge's geometry interpretation is fundamentally defective. Most pressingly their model relies on a robust distinction between "atomic" and "molecular" states of affairs, a distinction which I think is essentially foreign to Wittgenstein. I have described how this distinction can arise from a misapplication of the concept of mirroring to the distinction between an Elementsatz and a Satz. I have also presented the difficulty that the model faces with negated propositions, and outlined two possible extensions of the Goddard and Judge model to address this difficulty. The question of whether we can provide an interpretation of the Tractarian metaphysics together with later sections of the text remains an open and intriguing further direction to explore, one with ramifications on the consistency of the text, and hence on the debate between traditional and resolute readers of the Tractatus.

Acknowledgments

An enormous thank you to Paolo Mancosu for suggesting Goddard and Judge's book and geometric model in the wake of some initial, floundering looks into the philosophy of mathematics of the *Tractatus*, as well as continued help and discussion in forming the ideas and critiques of this paper. I owe him a huge debt of gratitude. Thank you also to Hans Sluga for guidance in the wide, wild sea of Wittgensteinalia and very useful help and discussion on a number of key points of this paper. And thanks to Sophie Dandelet and Ravit Dotan for very stimulating and fun initial brainstorming of ideas and positions, most of which thankfully never reached a wider audience, at cafes throughout Berkeley.

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